Suárez's Paradox and Contemporary Debates in the Philosophy of Quantum Logics

La paradoja de Suárez y los debates contemporáneos en la filosofía de las lógicas cuánticas

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ABSTRACT

In this commentary on Suárez's recently published work (2025), I aim to retrieve an original argument hitherto unpublished – which I refer to as *Suárez's Paradox* (Suárez, 1992) – together with the response he himself proposes. Drawing on both the paradox and the epistemological model proposed to resolve it, I argue that we are presented with a theoretical framework capable of enabling a philosophically significant approach to current debates on the interpretation of quantum logics. I conclude by assessing the value of this proposal not only in terms of its internal content but also considering its capacity to bridge two philosophical domains that, over recent decades, have largely evolved in isolation: the philosophy of logic and the philosophy of science.

KEYWORDS: Suárez's Paradox, Quantum Logics, Adoption Problem.

RESUMEN

En el presente comentario al ensayo recientemente publicado por Suárez (2025) busco recuperar un argumento original del autor, hasta ahora inédito, que he convenido en llamar *Suárez Paradox* (Suárez, 1992) y la respuesta que el propio Suárez propone. A través, tanto de esta paradoja como del modelo epistemológico propuesto para resolverla, argumento que estamos ante un marco teórico capaz de permitir una aproximación filosóficamente relevante a la discusión actual en torno a la interpretación de las lógicas cuánticas. Finalmente pondré en valor dicha propuesta atendiendo no solo a su contenido, sino también a cómo permite conectar los debates aislados acaecidos las últimas décadas en filosofía de la lógica y de la ciencia.

PALABRAS CLAVE: Paradoja de Suárez, Lógicas Cuánticas, Problema de la Adopción.

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1. Introduction

It is both an academic and a personal pleasure to offer this brief commentary on the recent publication of Professor Mauricio Suárez's M.Sc. dissertation, originally written in 1992 and only now, thirty years later, made public. From an academic standpoint, it is a genuine delight to see a text published in which a philosophically original thesis is defended – one that retains a remarkably fresh tone within current debates on theoretical questions that continue to revolve around the interpretation of quantum logics. My aim in this short piece is to draw attention to some of these questions and highlight their continued relevance.

But it is also, in a very real sense, a personal pleasure, shaped both by the motivations behind this publication and by the implications it bears. As for the motivations, they stem from intellectually stimulating conversations held in the Complutense's Faculty of Philosophy between late 2024 and early 2025 with Professor Mauricio Suárez, in which philosophical arguments were put forward with a lucidity capable of shedding new light on current discussions. As for the implications, these unfold on two levels.

On an objective level, the text contributes to bridging discussions that have historically taken place within the philosophy of science and those now occurring in contemporary philosophy of logic. On a more personal note, this publication has prompted a stimulating reorientation of my doctoral research and confirmed my conviction that philosophical inquiry into quantum logics goes well beyond the technical disputes among specialists in the formalism of orthomodular lattices.¹

The extensive new introduction that precedes Suárez's original text situates the work within its own history – from its initial composition to its eventual publication – and does so in the first person. For this reason, it would make little sense for me to attempt any further historical or contextual introduction. On the contrary, I believe it will be more fruitful for the reader if I highlight three

There is neither sense nor scholarly interest in offering here a detailed explanation of what are commonly referred to as "quantum logics". In the second section, I shall introduce them very briefly, with emphasis on a few philosophically relevant aspects rather than on technical ones. A thorough definition of the orthomodular lattices underpinning these logics can be found in Suárez (1992), to which I shall refer throughout – implicitly including the more introductory and expository material, to avoid unnecessary repetition. For readers seeking further background, I recommend the original article by Birkhoff and von Neumann (1936), the monograph Mittelstaedt (1978), or the opening sections of Bacciagaluppi (2013) among many others.

specific philosophical points of contemporary relevance, each of which features both in the original dissertation and in the new introduction.

This commentary revisits what I named Suárez's Paradox (SP) and defends the epistemological framework he proposes to resolve it. The model – based on a three-layer distinction between logic, theory, and empirical data – provides a powerful alternative to the logical monism shared by both Putnam's empiricism and Kripke's reply. Suárez's approach allows us to reinterpret the Adoption Problem and the status of quantum logics (QLs) in a way that bridges long-divided debates in the philosophy of logic and the philosophy of science.

2. Suárez's Paradox and the Three-Layer Model.

The expression *quantum logics* is, without doubt, polysemous. In its strict sense, the so-called *standard* quantum logics – to which I shall refer throughout as QLs – are algebraic structures derived from the analogue of classical *phase space* (Σ) within the formalism of quantum mechanics.² Just as in classical mechanics a *physical system* can be represented by assigning it a finite set of real numbers, typically three for position values and three momentum components, in quantum mechanics *physical systems* are instead represented by wave functions and projection operators on Hilbert space.

In the classical case, one may consider the set of so-called *experimental* propositions and represent it mathematically as the power set of the phase space over the reals – or, more precisely, as the subset of measurable combinations thereof.³ This operation, as is well known, yields a Boolean algebra by *Stone's* representation theorem. This algebra, in turn, admits a natural interpretation of its operations as logical connectives, allowing one to construct a standard classical propositional logic. In the quantum case, however, we do not work with the set of subsets of *Hilbert space* – the quantum analogue for the *phase space* –, but rather with the set of its *closed subspaces* $\mathcal{C}(\mathcal{H})$. The algebraic structure generated by this operation is no longer a Boolean one, but an orthomodular lattice – precisely

² Other uses of the expression include the study of quantum logical gates and circuits (see Dalla Chiara *et al.* 2018) or certain interpretations such as that of consistent histories (see Griffiths 2002).

³ If we have six-tuples as experimental propositions, $\langle r_1, ..., r_6 \rangle \in \mathbb{R}^6$, the total set of those will be equal to $\mathcal{P}(\Sigma)$ once we know that $\Sigma = \mathbb{R}^6$. If we want to deal with the more tractable set of all *measurable* subsets of the phase space – a proper subset of the power set – we will denote it by $\mathcal{F}(\Sigma)$.

because it possesses an orthocomplementation operation while relaxing the distributive property to its modular variant. Therefore, QLs emerge in response to the question of *how the algebraic operations of this structure may be interpreted logically*. The failure of distributivity ensures that we are no longer dealing with a Boolean algebra and, consequently, not with classical propositional logic.

It is precisely here that we encounter a wide range of proposals⁴ aimed at logically reconstructing the meaning of these algebraic operations through a well-defined syntax and semantics. And it is crucial that such reconstructions be coherent, for the failure of distributivity in the lattice does not seem to offer any insight into the interpretation of quantum experiments – experiments which, after all, have motivated the adoption of the Hilbert space formalism in the first place.

This is something Suárez (1992) himself highlights when he critiques a longstanding divergence within QLs that dates to their very origins in Birkhoff and von Neumann (1936). On the one hand, we have the formalism and the non-distributivity of the lattice; on the other, a range of heuristic and informal approaches that attempt to reconcile this non-distributivity with quantum experimental results – results which, at first glance, appear to challenge certain classical principles (though, as we shall see later, these may pertain more to a theory of probability than to logic proper). As Suárez remarks:

"In fact, the thought experiment seems totally unnecessary and inappropriate, in the context of LQM. The proof of the failure of the distributive law may be easily derived from the algebra of projectors, as shown on pages 22-23 in this dissertation. There is no need whatsoever for a thought experiment." (Suárez 1992: 38).

One further peculiarity of QLs, which rarely receives the attention it deserves, is precisely this: the fact that the lattice is non-distributive is one thing; that there should also exist a philosophical correlate capable of delivering epistemic gain by mapping that non-distributivity onto quantum results that resist interpretation under a classical ontology is quite another. The latter step is, in fact, an *ad hoc* addition.

And it was precisely this second step which, as Suárez (2025: 52) notes – following Bacciagaluppi (2013: 50) – motivated Putnam's (1968, 1974) claim that

⁴ These include approaches based on trivalent, n-valued, fuzzy, non-monotonic, intuitionistic, and substructural logics, among others. For a comprehensive overview of several major proposals, see Dalla Chiara and Giuntini (2002).

QLs might be of help in addressing the problem of interpreting the so-called *quantum paradoxes*. Bacciagaluppi (2013) offers several criticisms of Putnam's project and his allegedly empiricist proposal of a global revision of logic. However, a distinction must be drawn here. As Suárez (2025) rightly explains, Putnam's proposal is far from univocal and contains several important nuances – one of which is its continuity with Quinean empiricism. It is also crucial to note that Putnam's proposal is, above all, monist. It therefore makes little sense to reproach him for failing to develop a global revision in a pluralist sense – *a logic for each domain* – and thus the label *global* applied to his monist and anti-exceptionalist project ceases to be informative! There is, for Putnam, one correct Logic in the broad sense: the classical one.⁵

Putnam's idea, then, was to argue that certain logical principles – principles belonging to that classical logic – might have been thought of as *a priori*, and therefore as *necessary*. And yet, it might turn out that we could discover a theory – in this case, a physical one – within which a *different*, non-classical logic with alternative principles is found to be operative. The empiricist revision, then, would consist in preserving the existing logical framework *except* for those principles – here, distributivity – that we have discovered to be false. This notion rests on the *principle of the invariance of meaning* (Bell and Hallet, 1982). For Putnam, admitting this amounts to endorsing logical anti-exceptionalism and, accordingly, the empirical revisability of logic.

But as will become clear, this is a *sui generis* form of anti-exceptionalism, insofar as it is embraced from within a monist perspective. While it is true that logic would, on this view, be empirically revisable, the very sense of surprise and transgression involved in undertaking such a project reveals that logic nonetheless occupies a special status with respect to other disciplines. It is revisable, yes – but in a way that is neither obvious nor trivial.

⁵ Presumably, at the time of composing his series of articles, this referred specifically to first-order classical logic. I shall refer, interchangeably, to a relaxed version of standard classical logic, by which I mean either first-order logic (FOL) or second-order logic (SOL) supplemented by a standard axiomatic set theory (ZFC, BNGC, ...), as well as any equivalent formulations.

⁶ Given that for Putnam all *a priori* knowledge appears to be necessary – a highly questionable assumption; see Kripke (1972).

⁷ This is one of the distinctive aspects of QLs. Birkhoff and von Neumann (1936) themselves remarked on the strangeness of "discovering" a logic embedded within a theory that had not originally relied on that logic in its construction.

Putnam's idea, then, would be to propose an approach closely aligned with Quinean empiricism about logic: if such a "revision" or "improvement" of our logic were to be informed by quantum results, we might thereby be able to *dissolve* the so-called quantum paradoxes. For if something is properly defined in logical terms, it ceases – by definition – to be paradoxical, à la Frege, provided we respect certain minimal metalogical conditions.

For Bell and Hallet (1982), Putnam's idea was to salvage the classical metaphysics that had been abandoned by most interpretations of quantum mechanics – at the cost, however, of changing classical logic, which those very interpretations tended to retain (Suárez, 2025: 87). Bell and Hallet (1982) themselves showed that the principle of *meaning invariance* was technically unfeasible.⁸

Yet the possibility remained of upholding Putnam's position in the philosophy of logic from within a broadly monist, quasi-anti-exceptionalist framework. One could simply argue that quantum mechanics does not constitute the kind of informational source that could compel us to revise our one true Logic. All that would be needed is to take a step back and return to Quine. Kripke (2024)⁹ introduces his now-famous argument known as the *Adoption Problem* (AP) as a direct challenge to this possibility. From a similarly monist standpoint, Kripke accuses Putnam of attempting a *methodological ascent* in response to the interpretative problem posed by QLs. Analogous to how certain medieval scholastics might have proposed abandoning the language of reason in favour of the language of faith to think the divine ineffable from within finite minds, Putnam, in Kripke's view, proposes abandoning classical logic – reason – to grasp the ineffability not of God, but of a quantum physical system: *logica mechanicae quanticae ancilla est*.

But even setting aside the more controversial claims, Kripke places Putnam's proposal within a highly questionable characterisation – at least if one aims to remain committed to anti-exceptionalism. Kripke presents what he calls *Quine's Choice*: if I have a theory, and a hypothesis within that theory inferentially

⁸ Interestingly, for the same reason that the Kochen-Specker theorem blocks any hidden-variable interpretation: we cannot embed our QL into a larger Boolean algebra.

⁹ Curiously – just as in the case of Suárez (1992) – Kripke's (2024) text represents the posthumous publication of an argument first drafted decades earlier in response to Putnam's project of logical empiricism based on QLs. Moreover, as we shall see, the parallels between the two proposals – one situated in the philosophy of logic, the other in a more general frame – are striking, and they will serve, among other things, to insert several of Suárez's (1992) ideas quite naturally into the contemporary debate.

leads to a contradiction, I have two options: either remove the hypothesis or reject the inferential rules that led me to the contradiction. The latter appears to be Putnam's proposal, yet from a methodological standpoint it is highly implausible.

The AP, then, implies that even if we grant Quine's Choice – that is, even if we are prepared to adopt a new logic to make sense of a new result – we shall never actually be able to do so:

"Certain basic logical principles cannot be *adopted* because, if a subject already infers in accordance with them, no *adoption* is needed, and if the subject does *not* infer in accordance with them, no *adoption* is possible." (Padró 2024: 39).

Firstly, we may ask ourselves: How can logical monism – for both Putnam and Kripke – be sustained today, given the proliferation of well-established non-classical logics? The key lies in distinguishing Logic – with a capital 'L'¹⁰ or *logica utens*¹¹ – from what may be regarded as mere calculi, algebras, or non-classical logics as just formalisms:¹² these constitute *logica docens*. The crux of the matter is that the entire debate concerning whether classical *logica utens* is to be informed by QLs hinges on whether the latter are to be characterised as *docens* or not – or, of course, on the rejection of monism altogether. This same distinction is also implicit in van Fraassen (1974) and Suárez (2025), both of whom contrast algebraic, mere *docens*, operations with genuine propositions bound by logical principles such as bivalence, and thus truly *utens*.

In fact, Suárez (1992) adds a further dimension – one capable of offering a response without committing to logical monism, as we shall see shortly. This is the *three-layer model*, proposed as a response to *Suárez's Paradox*. He proposes that, prior even to the *utens/docens* distinction, one should adopt a purely epistemological framework of three layers. The first corresponds to *logica utens*; at least *epistemologically* in order to explain both the relation within other layers and this frame in philosophy of physics. The third – where the source of empirical information resides¹³ – can simply be called *world* or *reality*, in a *Tractarian* sense

¹⁰ Kripke (2024: 17).

¹¹ Birdman (2024: 45). The distinction *utens/docens* was a medieval one popularised in contemporary logic by Peirce (CP 1, III, 4.1.417, CP 2, I, 3.9.186-187 and CP 2, I, 3.10), where "CP" stands for Peirce (1931-1958)'s Collected Papers.

¹² Suzsko (1977) distinguishes between logic and algebras in the same spirit as *utens/docens*.

¹³ Suárez (2024).

- though, crucially, this remains an epistemological rather than ontological schema. The key lies in positing a second, intermediate layer: that of our *theories*. The task, then, is to determine where and how QLs are to be situated within this triadic model. An added benefit of this approach is that it safeguards the first layer from being collapsed into the others – thus preventing philosophical reflection from resting entirely on heuristics derived from natural language.

Suárez (1992: 88-89) then proposes the paradox we can summarize as follows:

- 1. The connection between phase space, Σ , and the observation space, ¹⁴ τ , is established [this is precisely what we have characterised as the standard QLs].
- 2. Elementary statements are, strictly speaking, *propositions* in the metaphysically bivalent sense¹⁵ [philosophically we can say in a *Tractarian* way, logically just in FOL sense].
- 3. Either truth values:
 - 3.1. Are dictated by the lattice of Σ [call it the class] $\mathcal{C}(\mathcal{H})$ or
 - 3.2. Dictate what the lattice is like, but not both.

The problem with insisting on both 3.1 and 3.2 is that we arrive at contradictions, and there is no third option. So, by *reductio ad absurdum* we must deny, at least, some of the premisses. Let me explore the contradictions before discussing whether to reject 1 or 2.

Let's reason by cases and suppose 3.1. Then we are saying that quantum propositions, as genuine metaphysical propositions, by 2, are *utens* and not a mere algebraic operationally formalism. But since semantical definitions ruling these propositions depend on $\mathcal{C}(\mathcal{H})$ operations, we must admit they are obtained empirically \hat{a} la Birkhoff and von Neumann (1936). So, on the one hand, they are independent of experience – since we impose well known restrictions from the quantum mechanics' formalism– and on the other hand they are not – since they are *utens* and thereof *a priori*. This option remarks the possibility that van Fraassen's notion of proposition – as used in SP –, linked directly to a *logica utens* based on bivalence among other principles – as used in Kripke's Adoption Problem – will be extracted by the QLs algebraic results. And this precisely leads

¹⁴ This is the technical way to introduce *quantum experimental propositions* in the $\mathcal{C}(\mathcal{H})$ formalism.

¹⁵ Following van Fraassen (1974).

to a meta-logical contradiction since we already defined this lattice using precisely this *utens* logic or, at least, a classical one.¹⁶

Let us now assume 3.2. is true. Then *utens* propositions dictate how orthomodular lattices in quantum domains are. But this is, again, something impossible. There is a well-known impossibility to generate a bigger Boolean lattice in which to embed quantum Hilbert. This would amount to hidden variables, which are ruled out by the Kochen-Specker theorem. Again 3.2. can be stretched out now in conditional form: if quantum propositions are *a priori* and therefore independent of experiments and empirical results, then FOL dictates the definition for Σ . This is the same as recognising that layer I determines layer III. In 3.1. logic depends on how the *world* is, but in 3.2 we obtain the inverse: the world depends on what logic we choose.

I have said that the contradictions revealed by SP resulted in the need to reject one of the two premises. Here we have three different options: (i) we can reject 1, (ii) reject 2 or (iii) both. The problem with (i) and, thereof (iii), is that 1 seems quite plausible. The connection between the quantum phase space and the so-called observation space is the definition of QLs –at least of strict QLs. This is because of the definition of proposition in QLs. Birkhoff and von Neumann considered that experimental propositions were the "subsets of the observation-spaces associated with any physical system" (1936: 824). This means that defining QLs using the elements of $\mathcal{C}(\mathcal{H})$ as their propositional basis serves to intrinsically connect both spaces (Suárez, 1992: 78). If we wish to distinguish between the propositional utens side of quantum propositions and the algebraic orthomodular docens side, we refer to 3.2. and 3.1., respectively. Thus, the plausibility of 1 undermines (i), and the dismissal of (i) in turn eliminates (iii). All that remains is (ii): deny the second premise that quantum propositions are strictly speaking propositions and not just elementary statements.

Thus Suárez's (1992) proposal is to attack 2 to make explicit that in 3's disjuncts we are discussing a direct connection from layer (I) to layer (III). And

¹⁶ In fact, the way we have properly defined the lattice itself is through a series of constraints on a classical structure: restricting distributivity by a modular law. And this first contradiction is what we may precisely call the "purported tension between QLs and classical logic" (Cuesta *et al.* 2025: 11).

¹⁷ In fact, when I introduced standard QLs – and pointed out different ways to approach them cf. note 15 below – those efforts can be read as an implicit defence of the mere possibility of 1. And it is also of no great interest to challenge the fundamental premise: that QLs are, at least, possible.

not just a mere connection, but a strict relationship of dependence in which one determines the other. But this means that we are ignoring that QLs are embedded into the theory in which we discovered them. Removing layer (II) is the main reason to obtain both contradictions and we have removed (II) in (2): "and this is the problematic assumption; for to accept propositions is to accept metaphysical bivalence, and bivalence is not the right metalogical principle to apply to orthomodular lattices" (Suárez, 1992: 89).

So far, so good. But how does this relate to the AP, and what insights does it offer for the philosophy of QLs? Two main points emerge. First, Suárez's Paradox bears a direct connection to the AP via van Fraassen's notion of *proposition* and the earlier notion of *logica utens*. This link, alongside the proposed three-layered epistemological model, opens the way for a renewed empiricist project – one that aligns with, yet remains more moderate than, Putnam's approach. This may be orientated in pragmatist terms once we have related *logica docens* with *theories* as a new layer. Second, and perhaps more provocatively, this framework allows us to reconsider the problem without presupposing a link between layers (I) and (III). This final move underlines the value of the proposed conceptual structure in revitalising the debate on QLs.

Suárez's three-layer model provides an elegant resolution to SP while reframing the AP in a way that avoids the logical monism of both Putnam and Kripke. By clarifying the theoretical role of QLs within scientific practice, it paves the way for a more pluralist and nuanced approach to the philosophy of logic.

3. Epistemological Lessons and Final Remarks

One of Suárez's (1992, 2025) most significant contributions lies in his refinement of Williamson's (2024) thesis: that one may accept a scientific theory without thereby adopting a new logic. Suárez qualifies this by noting that scientific theories, situated in the second epistemic layer, reflect inferential practices that may diverge from the first-layer logic but do not determine it. Classical logic remains stable and reliable as our logica utens, particularly in the construction of empirical scientific theories. Yet the model also opens space for pluralism: acknowledging classical logic as a privileged utens does not preclude alternative logics in other contexts, nor does it entail logical monism in general.

Indeed, Suárez's epistemological model offers a more precise framework than the traditional *utens/docens* dichotomy. It accounts for the historical proliferation of non-classical logics – intuitionistic, paraconsistent, of relevance, Cooper's logics, paraconsistent, etc. – motivated by our inferential practices in non-scientific

domains.¹⁸ The model allows these developments to be understood not as threats to logic's integrity, but as refinements conditioned by their epistemic context. It even accommodates more radical proposals that revise not just the logic of scientific theories, but also the set-theoretic or mereological assumptions of the first layer itself.

Importantly, this framework also defuses the long-standing operationalism vs realism debate. Once logic is restricted to the first layer and QLs are clearly situated in the second, the dichotomy dissolves: empirical theories need not dictate our logical principles, and metalogical reflection can proceed without being hostage to informal or heuristic interpretations. Suárez's model, in this sense, achieves what the Putnam-Finkelstein programme and Kripke's monist reply could not: a reconciliation between inferential pluralism and logical discipline.

Lastly, this triadic schema not only resolves SP but also facilitates long-overdue dialogue between the philosophy of science and the philosophy of logic. It grants a principled framework for distinguishing levels of inquiry, accommodating both classical and quantum perspectives without confusion or reductionism. Whether the reader accepts its implications or not, this proposal opens new paths for addressing the classical-quantum tension in the interpretation of QLs – paths that transcend the limitations of both logical monism and informal empiricism.

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¹⁸ Except, perhaps, for some substitutional programmes such as the intuitionistic or paraconsistent ones.

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