XIV JORNADA DE JÓVENES INVESTIGADORES/AS DEL I3A

Instituto Universitario de Investigación de Ingeniería de Aragón
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# Dynamic Consensus for Deformable Body Centroid Estimation

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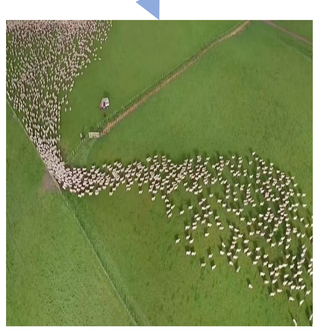
## Motivation

- Growing use of robots requires coordinated cooperation on shared tasks
- Robots must agree on key variables or functions to solve those tasks (consensus)
- Task may involve tracking deformable bodies (e.g. herding, disasters, industry)





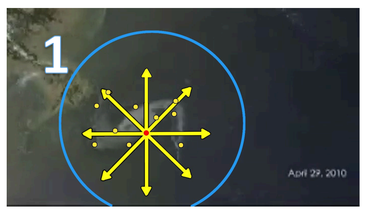
## Consensus Algorithm

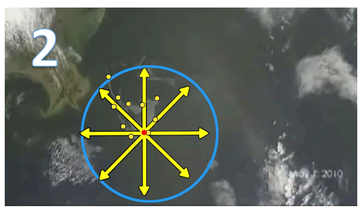
$$\dot{\boldsymbol{q}} = -\boldsymbol{L}\boldsymbol{q}$$

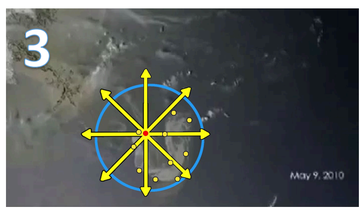
$$\dot{\boldsymbol{x}} = \alpha(\boldsymbol{x} - \boldsymbol{u}) - \boldsymbol{L}\boldsymbol{x} + \boldsymbol{L}\boldsymbol{q} + \dot{\boldsymbol{u}}$$

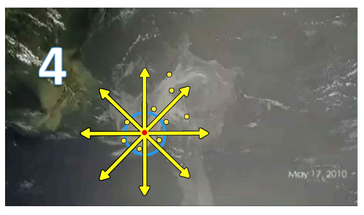
$$\boldsymbol{q}(t=0), \boldsymbol{x}(t=0) \in \mathbb{R}^{N\times 2}$$

$$\boldsymbol{q} = [\boldsymbol{q}_1, \dots, \boldsymbol{q}_i, \dots, \boldsymbol{q}_N]^T, \boldsymbol{x} = [\boldsymbol{x}_1, \dots, \boldsymbol{x}_i, \dots, \boldsymbol{x}_N]^T, \boldsymbol{u} = [\boldsymbol{u}_1, \dots, \boldsymbol{u}_p, \dots, \boldsymbol{u}_N]^T$$










Possible application: Tracking the BP 2010 oil spill. Despite oscillating tracked points (yellow), the estimated centroid (blue) converges towards the true centroid (red)

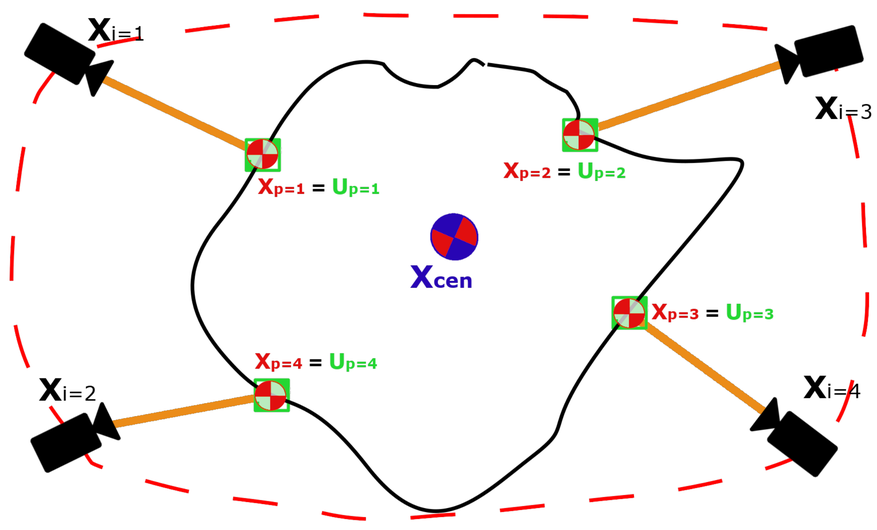
## Goal

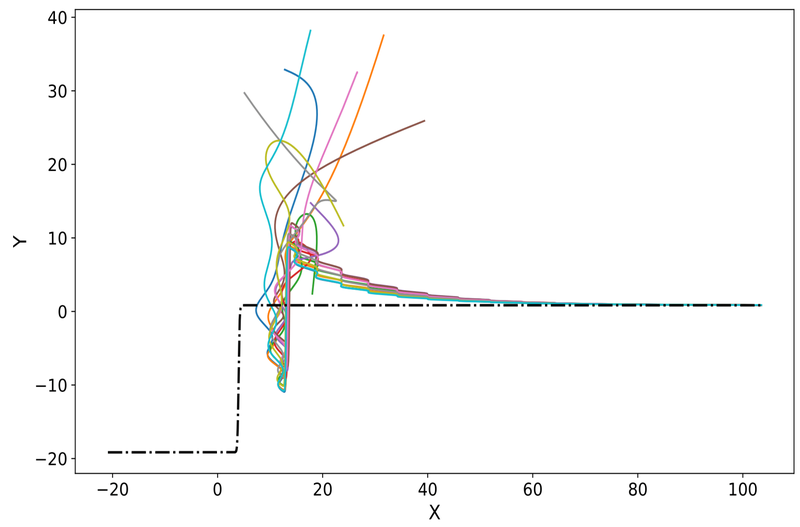
Use a **consensus algorithm** so the robots agree on the **centroid** of a **deformable body**

## Problem and methodology

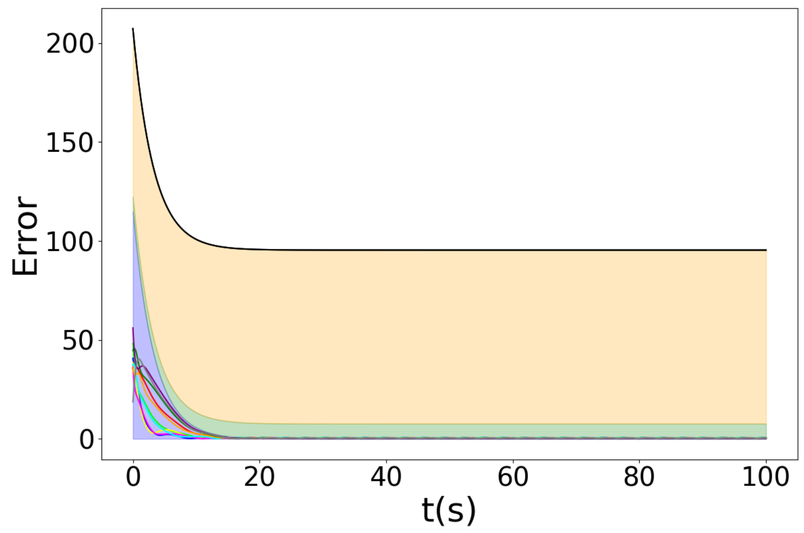
- $N$ agents estimate a centroid ($\boldsymbol{x}_{cen}$)
- Agents communicate with each other (graph)
- Agents track position of the points ($\boldsymbol{x}_p$) as functions ($\boldsymbol{u}_p$)
- Estimations agreement is consensus
- Points delimit the deformable body
- Points oscillate with amplitude $A$ (deformation)
- They communicate their centroid estimation ($\boldsymbol{x}_i$)
- Error is lower than a bounded maximum

$$|\boldsymbol{x}_{cen}(t) - \boldsymbol{x}_i(t)| = |\boldsymbol{e}_i^{deformable}| \leq |\boldsymbol{e}_i^{rigid}(t)| + \frac{\kappa||B||}{\lambda} 2\mathrm{A}(\mathrm{N}-1)$$

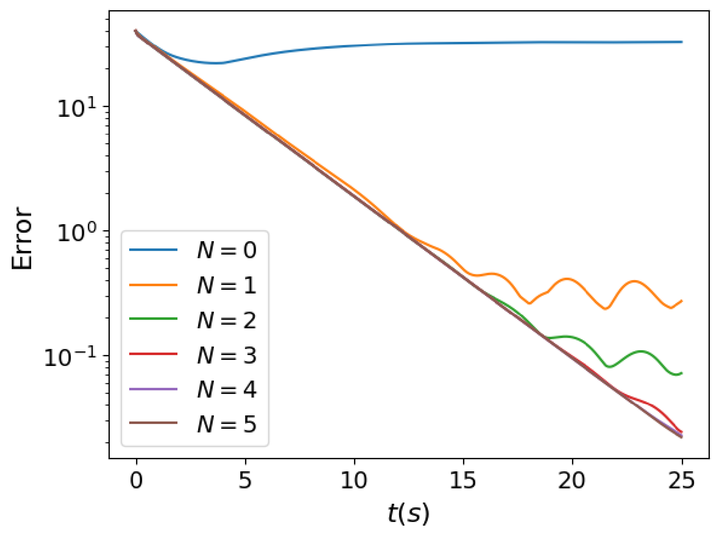



2D position of true (dashed) and estimated (solid) centroids over time




Experimental centroid error and maximum allowable bound for deformable bodies (black line)




Centroid estimation error vs number of agents each agent communicates with.

## Conclusions

- Applied Dynamic consensus to estimate centroid deformable body
- Computed a theoretical conservative maximum error bound

## References

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